XV. *On the Variations of the Daily Mean Horizontal Force of the Earth's Magnetism produced by the Sun's Rotation and the Moon's Synodical and Tropical Revolutions.* *By* J. A. Broun, *F.R.S.*

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History of the Investigation.

In a discussion of observations of the Bifilar magnetometer made at Makerstoun in 1844 a series of remarkable oscillations of the daily mean horizontal force were remarked, which, as the maxima occurred near new moon, and the minima near full moon, were attributed to the synodical revolution of our satellite*. This conclusion was afterwards confirmed by the result of four years' observations†. Kreil had found that the time of vibration of a horizontal needle was less near new moon and during the first quarter than near full moon and during the third quarter: this result he found only in the eight months November to June; while he obtained just the opposite result during the four months July to October‡.

In a much later investigation he found that his observations during the years 1842 to 1847 confirmed the result obtained by me, but that those for 1840 and 1841, and 1848 and 1849, were opposed to it, the magnetic force being greatest at full moon§. This change of law appeared to him connected with the decennial period.

An examination of observations made under my direction at Trevandrum in the years 1855 and 1856, and of other series, induced me to doubt whether the oscillations seen in the Makerstoun observations for 1844 could be explained by the lunar action. The appearance of maxima at full moon in similar oscillations in the observations of other years, the great amplitude of the oscillation and its nearly equal magnitude at different points of the earth's surface, induced me to seek for another cause. The only other apparent, with a period approaching that of the oscillation referred to, was the sun's synodic rotation on his axis. Kreil had already thought of this cause as an explanation of the different results obtained for the moon's action in summer and winter‖: this he considered might be due to the different solar poles presented to the earth at these two times of the year; but he soon perceived that any action due to the sun's rotation would be nearly the same in the means for the two seasons, since the

* Trans. Roy. Soc. Edinb. vol. xvi. part ii. p. 106 (read January 5, 1846).

† Ibid. vol. xix. part ii. p. xxxvi.

‡ Philosophical Magazine, vol. xvi. p. 241 (1840).

§ Wiener Denkschriften, B. v. S. 36 (1852). It was only after the results of the present paper had been obtained that I became acquainted with this memoir.

‖ Phil. Mag. vol. xvi. p. 241 (1840).

time required for a synodical rotation of the sun and revolution of the moon differs by upwards of two days, and the annual law of force obtained by him showed a maximum in June and a minimum in December. KREIL then relinquished this idea, and sought for the explanation in the difference of the meridian altitude of new moon, which is low in winter and high in summer, his results showing a greater horizontal magnetic intensity for the moon south than for the moon north of the equator. This latter result for the moon's declination did not accord with that afterwards obtained by me from the Makerstoun observations*.

Although an examination of the Trevandrum observations confirmed me in the belief that the sun's rotation was the great cause of the variations of daily mean horizontal intensity, yet trials for the greatest mean oscillation, employing periods of a few tenths of a day more or less than 27·2 days (the time given then by the best authorities for the sun's synodical rotation), were not satisfactory†, and the more complete investigation was deferred till the discussion of a longer series might enable me to draw more certain conclusions. Meanwhile, having found that the observations made at Trevandrum in 1844 and 1845 showed almost exactly the same variations as at Makerstoun, I returned to the examination of the observations made during these two years, and deduced from them, and from the observations made simultaneously in different Colonial observatories, that the mean duration of the oscillation was 25·96 days‡. This period differed so much from that then accepted for the sun's rotation that I suggested some movement, regular or irregular, of the sun's magnetic poles or of the meridian of maximum force §.

In 1871 Dr. HORNSTEIN deduced from his observations at Prague in 1870, by calculation and graphic interpolation, a period 26·33 days, but varying from 26·0 to 26·7 days according to the instrument or method employed in the investigation. The mean period agreed almost exactly with that which had then been obtained by SPOERER for the sun's synodical rotation from spots in the equatorial zone. This result, brought to the notice of the Royal Society, Nov. 16, 1871, by the Foreign Secretary, and referred to in two presidential addresses, was the subject of a note in the 'Proceedings' of the Society (vol. xx. p. 417, June 1872). In a postscript to this note the following passage occurs:—"In my paper on the Horizontal Force of the Earth's Magnetism, I stated that 'a careful investigation of a much larger series of observations leads me to believe that the period is variable within certain limits' ||; whether this variation is due to a change

* Trans. Roy. Soc. Edinb. vol. xix. pt. ii. p. xxxvi.

† Writing from memory, I have previously said "26 to 27 days" were tried; this was a mistake. Proc. Roy. Soc. xx. p. 418 (1872).

‡ Trans. Roy. Soc. Edinb. vol. xxii. p. 543.

§ Phil. Mag., Aug. 1858. I think there is little doubt that KREIL would have discovered the variations of the earth's magnetism due to the sun's rotation if he had not believed that period to occupy nearly the same time as the moon's tropical revolution. My own belief that the oscillations observed by me were due to this cause in spite of the difference of periods, was founded on the comparison of observations in both hemispheres, where the agreement seemed inexplicable by the lunar action alone.

|| Trans. Roy. Soc. Edinb. vol. xxii. p. 544.

of the solar meridian producing the maximum after certain intervals, or to superposed regular or irregular causes, I hoped, and hope yet, to determine."

One of the most marked objections to the constancy of the solar action for given solar meridians for any considerable period was to be found in the middle of the series of well-marked oscillations for 1844. In the months of January to April there were four successive movements with periods of from 25 to 29 days, each having amplitudes of about one thousandth of the whole horizontal force; in the months of May, June, and July the oscillation may be said to have disappeared, while in the following months the movement reappeared as distinctly as in the first months of the year. It is the principal object of the present communication to show that this apparently anomalous result is due to the joint actions of the sun and moon.

Solar and Lunar Actions.

On a reconsideration lately of the results obtained by me previously for the moon's synodical revolution from four successive years' observations at Makerstoun, it seemed extremely improbable that they could be wholly explained by an action having a period of 26 days. Since in a discussion of a year's observations nearly 14 periods of 26 days and 12 of 29·53 days are employed, if the solar action remains the same for each period the maximum must have happened at nearly all the positions of the moon, and should have an equal effect at all epochs in the discussions relatively to the revolution of our satellite. If the moon's position had no effect on the magnetic variation, any result which might be obtained could be due only, as was supposed in this case, to irregularity in the solar action, and would certainly be different in the discussion of observations made in successive years. Some difference there was without doubt; but a reconsideration of the whole results has induced me to conclude that the agreement was too great to be explained by any other hypothesis than that of a conjoint action of the sun and moon.

The simplest method of determining the truth of this supposition was the following:—To obtain from the observations for a given time the mean variations due to each of three periods (namely, that of 26 days supposed due to the solar rotation, and those of 29·53 and 27·3 days due to the lunar synodical and tropical revolutions), to assume that these mean results represent the solar and lunar actions during each period (or that the action of each body remains constant for the same position) throughout the whole number of periods included in the investigation, and to obtain the variations of horizontal force for each day of the year by their addition. Such a construction will show to what extent the assumption is well founded.

In order, however, that this construction may be as accurate as possible, the mean results obtained for each body must be free from the effects due to the others. For this investigation we have the following equalities:—

Lunar synodical period	15×29·53=442·95 days.
Solar	17×26·0 =442·0 ,,
Lunar tropical	20×27·3 =546·0 ,,
Solar synodical	21×26·0 =546·0 ,,
Lunar synodical	12×29·53=354·36 ,,
Lunar tropical	13×27·3 =354·9 ,,

As the projected observed daily means of the horizontal force for 1845 show a more frequent action of irregular disturbing causes than those for 1844, the discussion has been made separately for the two years.

In the calculations for the lunar synodical and tropical periods, 12 of the first and 13 of the second were taken; as these occupy the same time, the effects due to the one are eliminated in the calculation for the other (constancy of law being always assumed). This is not the case, however, with reference to the solar period of 26 days, for which 14 periods were taken.

Since it requires 17 and 21 periods of 26 days for the elimination of the effects due to the lunar synodical and tropical periods respectively, if we seek the mean result for the solar rotation from 14 periods (=364 days) corrections must be applied on account of lunar actions not eliminated. Let $1'$, $2'$, $3'$ . . . . $26'$ represent the mean values of the horizontal force for the 1st, 2nd, ... 26th days in the solar period; $1''$, $2''$, $3''$, ... $29''$, $30''$ represent those corresponding to the days of the lunar synodical action, and $1'''$, $2'''$, ... $27'''$ those for the lunar tropical action. If we neglect the variations due to other causes, the mean horizontal force for each day, commencing all the periods with the 1st January 1844, may be represented as follows:—

Number of solar period.	1st day.	2nd day.	26th day.
1.	$1'+1''+1'''$	$2'+2''+2'''$	$26'+26''+26'''$
2.	$1'+27''+27'''$	$2'+28''+0'''{\cdot}7$	$26'+22''{\cdot}5+24'''{\cdot}7$
⋮	⋮	⋮	
14.	$1'+14''{\cdot}5+11'''{\cdot}4$	$2'+15''{\cdot}5+12'''{\cdot}4$	$26'+10''+9'''{\cdot}1$
15.	$1'+11''+10'''{\cdot}1$	$2'+12''+11'''{\cdot}1$	$26'+6''{\cdot}5+7'''{\cdot}8$
16.	$1'+7''{\cdot}5+8'''{\cdot}8$	$2'+8''{\cdot}5+9'''{\cdot}8$	$26'+3''+6'''{\cdot}5$
17.	$1'+4''+7'''{\cdot}5$	$2'+5''+8'''{\cdot}5$	$26'+29''{\cdot}5+5'''{\cdot}2$
⋮	⋮	⋮	
20.	$1'+24''+3'''{\cdot}6$	$2'+25''+4'''{\cdot}6$	$26'+19''{\cdot}5+1'''{\cdot}3$
21.	$1'+20''{\cdot}5+2'''{\cdot}3$	$2'+21''{\cdot}5+3'''{\cdot}3$	$26'+16''+0'''$

If now we take the sums for 17 periods of the 2nd term in each day, we may represent it in each case by

$$2\Sigma^{1}_{29\cdot5}L_{S'}=C,$$

twice the sum of the lunar synodical variation as deduced from 17 successive means at intervals of $3\frac{1}{2}$ days. In the same way the sums for 21 periods of the 3rd term in each day gives

$$\Sigma^{1}_{27\cdot3}\mathrm{L_T}=\mathrm{C}',$$

the sum of the lunar tropical variations derived from 21 successive means at intervals of 1·3 day. If we employ, as in the following investigation, instead of the daily mean horizontal force the variations of these values from their mean, the constants C, C′ will become equal to zero. On taking then the means of the three terms for each day in 14 periods, we obtain

$$\text{1st day} \; . \; . \; . \; . \; . \quad 1'-\frac{11''+8''+4''}{14}-\frac{10'''+9'''+8'''+6'''+5'''+4'''+2'''}{14},$$

$$\text{2nd day} \; . \; . \; . \; . \quad 2'-\frac{12''+9''+5''}{14}-\frac{11'''+10'''+9'''+7'''+6'''+5'''+3'''}{14},$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\text{26th day} \; . \; . \; . \; . \quad 26'-\frac{6''+3''+29''}{14}-\frac{8'''+7'''+5'''+4'''+3'''+1'''+0'''}{14}.$$

The *corrections* of the means for the 1st, 2nd, ... 26th days of the solar rotation derived from 14 periods are the quantities given in the 2nd and 3rd terms, with the signs changed.

In a similar manner the *corrections* of the lunar synodical and tropical variations derived from 12 and 13 periods respectively on account of the solar action uneliminated are for the lunar synodical period—

$$\text{1st day} \; . \; . \; . \quad +\frac{17'+21'+24'}{12},$$

$$\vdots \qquad\qquad \vdots$$

and for the lunar tropical period—

$$\text{1st day} \; . \; . \; . \quad +\frac{18'+19'+20'+22'+23'+24'+26'}{13},$$

$$\text{2nd day} \; . \; . \; . \quad +\frac{19'+20'+21'+23'+24'+25'+1'}{13},$$

$$\vdots \qquad\qquad \vdots$$

The preceding expressions represent the corrections when all the periods are supposed to commence with the same day; this, however, was not the case in the following calculations:—

For 1844,	the solar period commences	1	January.	
,, ,,	the lunar synodical period commences .	5	,,	(full moon).
,, ,,	the lunar tropical period commences .	3	,,	(moon furthest north).
For 1845,	the solar period commences	30	December, 1844.	
,, ,,	the lunar synodical period commences .	24	,, ,,	
,, ,,	the lunar tropical period commences .	23	,, ,,	

Hence, in 1844, $4''$ has to be subtracted from the lunar synodical and $2'''$ from the lunar tropical corrections of the solar period, while in the corrections of the former for the latter $4'$ and $2'$ respectively must be added. In 1845 the corresponding changes are $+6''$ and $+7'''$ in the first cases, and $-6'$ and $-7'$ in the last.

Tables I., II., and III. contain the mean variations for each of the three periods in the years 1844 and 1845 as derived from the means of 24 hourly observations in each *civil day*, together with the corrections obtained by the preceding expressions and the corrected means. It should be remarked that the quantities represented by $1'$, $2'$, ... $1''$, $2''$, ... $1'''$, $2'''$, ... have each been supposed unaffected by the actions of other periods; in seeking these corrections, however, the uncorrected means as derived from 14, 12, and 13 of the respective periods were employed. Any error due to this cause might have been avoided by a repetition of the corrections; the error, however, was found to be negligible in this investigation.

If we could assume not only that the variations found are those produced by the sun and moon in each period of their respective rotations or revolutions during twelve months, but also that there are no other causes of variation, we might then construct the observed results by the addition of the three variations due to the positions of a given solar meridian and of the moon relatively to the earth and sun on each day of the year. All notice, however, has been omitted of other regular or irregular causes of variation; and we have only to look at the projections of the observed means to see that great changes of the mean horizontal force occur within the space of one or two days, at intervals which cannot be connected with any continuous law. Frequent disturbances appear, especially in the means for the year 1845 (see Plate 38). It might be allowable, then, to omit these means in the investigation, or to employ interpolated quantities in their stead. It is difficult, however, to fix on the limit beyond which means should be excluded, without giving some grounds for the idea that the results obtained might not have been the same had all the observations been included. I have thought it best under these circumstances to employ *all* the observations in obtaining the variations given in Tables I., II., and III., and to make no attempt to "improve" the appearance of the results by any artifice. The year 1844 was one of minimum magnetic disturbance, and on this account the better fitted to show any law which is independent of these apparently irregular variations; I shall for this reason consider the results for that year first.

Having entered the variations belonging to the solar period in columns corresponding to each day of the year, the first quantity (column 5, Table I.) appeared opposite January 1, January 27, February 22, and so on, repeated 14 times; the quantities in column 5 of Table II. were placed in the next column, so that the first was opposite January 5, February 4 ..... (days of full moon), the remaining quantities following in order for each synodical revolution. Similarly the first variation (column 5 of Table III.) was entered opposite January 3, 30, February 27 .... (days for which the moon's north declination was greatest), the other variations following in their order. The sums of the three quantities were then taken for each day of the year.

TABLE I.—Variations of Daily Mean Horizontal Force for the Solar (26-day) period in millionths of the whole Horizontal Force.

Solar Day.	1844. Observed Solar.	1844. Corrections for Lunar: Synodical.	1844. Corrections for Lunar: Tropical.	1844. Corrected Solar.	1845. Observed Solar.	1845. Corrections for Lunar: Synodical.	1845. Corrections for Lunar: Tropical.	1845. Corrected Solar.
1.	+146	−44	+35	+137	+ 49	+ 7	+21	+ 77
2.	+157	−36	+23	+144	−376	−10	+16	−370
3.	+105	−28	+13	+ 90	−111	+13	−16	−114
4.	+172	−16	− 8	+148	− 28	+11	−10	− 27
5.	+ 66	−13	−16	+ 37	− 12	−13	− 4	− 29
6.	+138	− 1	−19	+118	+ 89	+ 6	+11	+106
7.	− 23	+ 8	−17	− 32	+109	+ 6	− 7	+108
8.	+ 89	+23	−11	+101	+ 70	−31	−19	+ 20
9.	−128	+29	− 2	−101	+149	−12	−18	+119
10.	−105	+32	+ 9	− 64	−281	+ 3	+ 1	−277
11.	−214	+38	+11	−165	−307	− 8	− 7	−322
12.	−324	+41	+12	−271	−194	− 8	+20	−182
13.	−318	+46	+ 6	−266	−157	+15	+17	−125
14.	−259	+46	+ 4	−209	− 93	+10	+26	− 57
15.	−313	+45	−14	−282	− 47	−21	+25	− 43
16.	−248	+46	−17	−219	+ 23	+ 6	+22	+ 51
17.	−110	+44	−26	− 92	+113	+17	+10	+140
18.	− 95	+35	−35	− 95	+ 85	− 3	+13	+100
19.	+ 8	+21	−43	− 4	+167	− 2	− 4	+161
20.	+ 62	+13	−38	+ 37	+269	+ 4	−12	+261
21.	+ 30	+ 2	−23	+ 9	+214	− 7	−22	+185
22.	+197	− 8	+ 2	+191	+ 6	− 9	− 9	− 12
23.	+307	−22	+18	+303	+ 65	+ 3	−26	+ 42
24.	+254	−26	+33	+261	+154	−10	−23	+121
25.	+267	−29	+31	+289	+ 49	−14	−20	+ 15
26.	+145	−42	+35	+138	+ 15	+13	−11	+ 17

NOTE.—The observed quantities (Tables I., II., & III.) are deduced from the means for each civil day given in Table xxii., Makerstoun Observations, Edinb. Trans. vol. xviii. p. 355, and Table xvii. vol. xix. pt. 2, p. 11, the means being corrected for secular change at the rate of −0·73 *per diem* for 1844, and of −0·25 for 1845. The order of corrections for the different Tables was as follows:—In 1844, 1st, the lunar tropical variations were corrected for the solar variations, the means of the variations for each three solar days being taken as corresponding to the middle day before obtaining the correction; 2nd, the solar variations were corrected for the lunar tropical (means of three taken); 3rd, the solar were corrected for the lunar synodical; 4th, the lunar synodical were corrected for the solar. In 1845, the 1st and 2nd as in 1844, the 3rd of 1844 was last in 1845.

TABLE II.—Variations of Daily Mean Horizontal Force for the Lunar Synodical Period (29·5 days), commencing at full moon, in millionths of the whole Horizontal Force.

Day.	1844.			1845.		
	Observed Lunar.	Corrections for Solar.	Corrected Lunar.	Observed Lunar.	Corrections for Solar.	Corrected Lunar.
1.	−208	+37	−171	+ 52	+ 1	+ 53
2.	−169	+35	−134	+ 42	+ 1	+ 43
3.	−305	+45	−260	− 7	+14	+ 7
4.	−221	+37	−184	− 45	+17	− 28
5.	−341	+41	−300	− 99	+20	− 79
6.	−138	+21	−117	+ 26	+26	+ 52
7.	− 56	+23	− 33	−259	+25	−234
8.	− 21	+13	− 8	−161	+21	−140
9.	− 40	0	− 40	+ 60	+22	+ 82
10.	+111	+ 7	+118	+109	+14	+123
11.	+185	−28	+157	−107	+ 5	−102
12.	+151	−18	+133	+122	− 6	+116
13.	+265	−34	+231	+162	− 8	+154
14.	+209	−38	+171	+148	− 5	+143
15.	+259	−49	+210	+ 2	− 5	− 3
16.	+266	−30	+236	+108	+ 6	+114
17.	+145	−45	+100	− 67	+ 1	− 66
18.	+205	−41	+164	−105	− 9	−114
19.	+179	−27	+152	+ 59	−21	+ 38
20.	+267	−25	+242	+102	−19	+ 83
21.	+231	− 8	+223	−123	−19	−142
22.	+148	+10	+158	− 41	−18	− 59
23.	+165	+14	+179	−259	−21	−280
24.	+169	+32	+201	− 57	−11	− 68
25.	−106	+38	− 68	+ 83	− 9	+ 74
26.	−274	+38	−236	+ 50	−12	+ 39
27.	−211	+37	−174	+146	−12	+134
28.	−333	+35	−298	− 40	+ 1	− 39
29.	−222	+45	−177	+145	+14	+159
30.	−313	+37	−276	− 53	− 7	− 60

NOTE.—When there were only 29 days in a lunation, −227 (the mean for the 29th and 30th days) was employed for the 30th day in 1844, and +50 in 1845. See Note to Table I.

TABLE III.—Variations of Daily Mean Horizontal Force for the Lunar Tropical Period (27·3 days), commencing with Moon's greatest North Declination, in millionths of the whole Horizontal Force.

Day.	1844.			1845.		
	Observed Lunar.	Corrections for Solar.	Corrected Lunar.	Observed Lunar.	Corrections for Solar.	Corrected Lunar.
1.	+179	+ 83	+262	+177	−29	+148
2.	+164	+ 92	+255	+ 57	− 1	+ 56
3.	+269	+ 95	+364	+136	+19	+155
4.	+ 88	+ 97	+185	−124	+40	− 84
5.	− 61	+ 84	+ 23	− 3	+54	+ 51
6.	−263	+ 74	−189	− 75	+62	− 13
7.	−172	+ 54	−118	−132	+59	− 73
8.	−145	+ 45	−100	−232	+62	−170
9.	− 65	+ 23	− 42	−154	+53	−101
10.	− 61	+ 9	− 52	− 99	+41	− 58
11.	− 75	− 20	− 95	−130	+13	−117
12.	+116	− 39	+ 77	+ 29	− 1	+ 28
13.	+ 67	− 68	− 1	− 47	−14	− 61
14.	+151	− 86	+ 65	− 50	− 9	− 59
15.	+130	−106	+ 24	+129	−10	+119
16.	+190	−114	+ 76	+163	− 7	+156
17.	+202	−120	+ 82	+166	−22	+144
18.	+129	−116	+ 13	+ 5	−20	− 15
19.	+ 73	−103	− 30	−328	−23	−351
20.	+ 19	− 81	− 62	− 38	−20	− 58
21.	+ 13	− 55	− 42	− 1	−32	− 33
22.	−169	− 25	−194	+ 48	−36	+ 12
23.	−120	+ 7	−113	+ 80	−38	+ 42
24.	−103	+ 36	− 67	+ 24	−37	− 13
25.	−284	+ 62	−222	+ 67	−50	+ 17
26.	−134	+ 75	− 59	+161	−46	+115
27.	−125	+ 83	− 42	+182	−29	+153

NOTE.—When there was a 28th day in the period, +110 and +150 (the means for the 27th and 1st days) were employed for that day in 1844 and 1845 respectively. See Note to Table I.

It will be remembered that these variations are those about the means in each case; it has been considered a sufficiently near approximation to the mean horizontal intensity for each day, and for each of these periods, to take the means for four weeks, two weeks preceding and two weeks following each day. The observed means for each *civil* day having then been projected and connected with black lines, the curve of four-weekly means was formed by a dark dotted line (see Plate 38); from this line the sums of the three variations were set off *plus* or *minus*, and connected by red lines drawn from point to point.

It requires only a glance at these projections to see with how much fidelity the calculated represents the observed curve, in spite of the effects of the larger irregular variations on the means for each period, and on the appearance of the observed curves for the periods in which they occur.

The conclusions drawn are as follows:—

1st. The increase of amplitude of the oscillations which appears in February and March, and in September to November, 1844, is explained by the occurrence, near the same time, of the maximum actions of the sun and moon. In order to show this fact more distinctly, the separate actions of each body are projected above the oscillation for February 1844 (Plate 38).

2nd. The diminution and apparent disappearance of the oscillations in June and July, which seemed so inexplicable when only one cause of variation was supposed to exist, are explained by the opposing actions of the two causes, the maximum of the one occurring at the same time as the minimum of the other. The independent actions of each body are shown in the projections below the movement for June (Plate 38).

3rd. The variation of the time of single oscillations is explained in a similar manner by the combinations of the different variations.

4th. Since the agreement between the observed and calculated results is so considerable, we are entitled to conclude that during a period of twelve months the solar and lunar actions remained nearly constant for the same positions of these bodies relatively to the earth and to each other.

Had it been sought merely to establish the fact that the variations of the daily mean horizontal force of the earth's magnetism may be represented very nearly in years of small magnetic irregularity by the computed variations due to three causes having periods of 26, 27·3, and 29·5 days, the investigation might have stopped here. It seemed to me, however, desirable to examine to what extent the same process would succeed in a year for which the movements appeared more irregular. The variations for 1845 were therefore calculated from the means in the last columns of Tables I., II., and III., in the same way as has been explained for 1844, commencing January 1, 1845, with the 3rd day of the solar, the 9th of the lunar synodical, and the 10th of the lunar tropical period.

The sums of variations for each day having been obtained, they were set off from the curve of four-weekly means as before (see Plate 38).

We perceive at once that the *observed* variations of daily mean horizontal force did not show the same regularity in the oscillations as in 1844; considerable disturbances seem to have occurred at irregular intervals destroying the symmetry of the movements. It could scarcely be expected that any calculation in which the same quantities are employed successively from 12 to 14 times during the year could give an approximation to the observed movements throughout the period. I believe, however, it will be seen that the resemblance of the calculated and observed results is very considerable; and although at first sight the agreement of the red and black oscillations may not appear so satisfactory as in 1844, yet a more minute examination will show the frequent coincidences, which are all the more extraordinary that the irregularities throughout the year appear so marked. The variations due to each period in 1845 are projected below the movements for June. The maximum solar action occurs about 4 or 5 days earlier in 1845 than in 1844, the supposed duration of the period being 26 days; this may be due to a change of the solar meridian producing the maximum, or to the action of superposed irregular disturbances (which, there can be no doubt, have an effect on the mean result), or to the true period being slightly less than 26 days*.

The lunar action has also undergone a change, since a maximum occurs both near new and near full moon; this result is also affected by the irregular variations. I shall notice immediately the fact that the moon's action in producing the magnetic variations appears to depend upon the sun. There is in 1845, as in 1844, a maximum both when the moon is near her greatest northerly and her greatest southerly declination.

It is not my object here to enter into the differences of the results for each period as derived from each of the two years' observations, or to seek whether these differences may not be due to the greater number of irregular movements which occurred in 1845; the consideration of these questions must include the study of a longer series of observations; yet I may add that it seems to me the least probable supposition that the meridian of maximum solar action which remained constant throughout 1844 had changed in 1845.

The conclusion deduced from the curves for 1844, of the constancy of the solar and lunar actions during twelve months, is confirmed generally, I think, even with the superposed sudden movements of 1845.

Magnetic Pulsations.

The view generally entertained of the source of the great magnetic disturbances is, that they are produced by some action of the sun; this conclusion is founded on the diurnal law of frequency and magnitude which these movements follow. The diurnal law, however, appears modified by conditions of latitude; the maximum disturbance which occurs near midnight in high latitudes happens near noon within the tropics. I have shown, however, that the great changes of *daily mean* horizontal intensity are experienced similarly at all the stations considered between 56° North and 42° South

* See Proceedings of the Royal Society, vol. xx. p. 421.

latitude where observations had been made*. When a great diminution of mean daily force occurs at one station, a similar and nearly equal diminution occurs at the others. If this diminution be due to a solar action only, such as, it has been supposed, might result from some considerable movements in the sun's envelopes, and if the moon has no part in the production of the resulting changes of terrestrial magnetism, the means obtained for the solar period will be affected by true solar actions, while those for the lunar periods would be affected by changes in the production of which the moon took no part. Were this certain there would be reasons for omitting great and sudden departures from the mean in the discussions for the moon's periods, which do not exist in those for the sun. As the mean variations due to the moon's action appear so considerable when compared with those due to the sun, it does not seem at all certain that the great magnetic disturbances are wholly solar. An effort is here made to examine this subject.

We have seen that when one side of the sun is presented to the earth, the magnetic force of the latter is greater than when the other side is turned towards us; we may even say that the intensity is greatest for a given solar meridian; this, however, may be simply an integral effect resulting from the actions due to all the meridians. But can we suppose when a great and sudden increase or diminution of the earth's magnetic force occurs that this is produced by some change occurring on a particular solar meridian? This does not seem at all improbable. It could scarcely, however, be supposed that the effect produced on the earth's magnetism could be due to the given solar meridian being presented to (that is, in the same plane with) the earth, unless we admit the idea that these solar magnetic actions are propagated only in certain definite directions, or that the earth has some action in the production of the solar change when the given meridian is presented to it.

In order to examine the facts, all the cases were noted during the years 1844 and 1845 in which the *daily mean* horizontal force diminished one thousandth of its whole value *within* an interval of three days; they were found to be twenty-eight in number. If we call the solar meridian presented to us on the 1st January, 1844, the zero meridian (0), and consider the time of rotation to be 26 days, and that there are 26 meridians, we find that the solar meridians presented to us when these great movements occurred may be arranged in a few groups, as in the following Table.

* Trans. Roy. Soc. Edinb. vol. xxii. p. 545.

TABLE IV.—Cases in which the Earth's Magnetic Force diminished one thousandth of its whole value or more in 1844 and 1845.

No. of case.	Dates of beginning and ending.	Change of force in hundred thousandths.	Solar meridians.		
			5 to 10.	11 to 14.	−3 to +1 and others.
1.	1844. Mar. 28 to Mar. 30	−360	+8 to +10		
2.	Apr. 25 „ Apr. 26	104	....	+11 to +12	
3.	May 21 „ May 23	107	....	+11 „ +12	
4.	July 8 „ July 9	116	+7 to + 8		
5.	Aug. 1 „ Aug. 2	175	+5 „ + 6		
6.	Aug. 9 „ Aug. 10	135	....	+13 „ +14	
7.	Sept. 24 „ Sept. 27	115	+7 to +10		
8.	Sept. 30 „ Oct. 1	104	....	+13 „ +14	
9.	Oct. 19 „ Oct. 21	268	+6 to + 8		
10.	Nov. 19 „ Nov. 21	130	....	+13 „ +14	
11.	Dec. 27 „ Dec. 30	165	....	....	−3 to 0
12.	1845. Jan. 8 „ Jan. 10	210	+9 to +11		
13.	Jan. 18 „ Jan. 20	163	....	....	(−6 „ −4)
14.	Feb. 20 „ Feb. 21	118	....	....	0 „ +1
15.	Apr. 12 „ Apr. 14	350	....	....	−1 „ +1
16.	June 10 „ June 12	110	+6 to + 8		
17.	July 24 „ July 25	154	....	....	−2 „ −1
18.	July 31 „ Aug. 3	102	+6 to + 9		
19.	Aug. 26 „ Aug. 27	101	+5 „ + 6		
20.	Aug. 29 „ Aug. 30	159	+8 „ + 9		
21.	Sept. 24 „ Sept. 25	153	+8 „ + 9		
22.	Oct. 8 „ Oct. 10	126	....	....	(−4 „ −2)
23.	Oct. 20 „ Oct. 21	118	+8 to + 9		
24.	Oct. 30 „ Nov. 1	100	....	....	(−8 „ −6)
25.	Nov. 15 „ Nov. 18	100	+8 to +10		
26.	Nov. 27 „ Nov. 29	110	....	....	(−6 „ −4)
27.	Dec. 3 „ Dec. 4	291	....	....	0 „ +1
28.	Dec. 11 „ Dec. 13	122	+8 to +10		

An examination of this Table will show that nearly half of the great changes began when the 8th meridian after the zero had passed, while five began near the 12th after, and five near the zero itself. The two greatest changes of magnetic force in 1844 happened near the + 8 meridian, and the two greatest in 1845 at the 0 meridian*.

* KREIL, who obtained so many interesting results, has noticed the fact that great disturbances occurred frequently on the same, or nearly the same, day of successive years. "Thus the greatest disturbances which took place in the year 1836 were on the 22nd and 23rd April and on the 18th October, and both these were repeated on the same days in 1837." In 1838 several disturbances occurred within a few days of the same dates of 1837, a list of which he gives. He also draws attention to the fact of "the symmetrical arrangement of these disturbances in the same year, many of them being nearly six months apart" (Phil. Mag. 1840, vol. xvi. p. 246). It has been pointed out that 14 solar rotations are performed in 364 days, or within one day of the year; there are also 7 rotations in six months. The recurrence exactly after twelve or six months does not appear for the years 1844 and 1845, but of the five *greatest* changes of daily mean horizontal force (Cases Nos. 1, 9, 12, 15, and 27) the first two were at an interval of 8 periods of 26 days, the second and third at an interval of 3 periods of 26 days, and the fourth and fifth at an interval of 9 solar rotations. As, however, the disturbances obey also an annual law with the maxima near the equinoxes (as I first pointed out in 1847), the probability is the greatest of disturbances recurring at the end of 7 rotations (six months), or 12 rotations (one year), near the equinoxes.

If any doubt existed as to the possibility of these being mere accidental coincidences, it would be removed, I think, by a consideration of the marked succession occurring between July 31st and December 11th, 1845. It will be observed that though considerable magnetic changes occur successively near the same solar meridian, yet that they do not always occur in successive rotations; thus a great disturbance commenced at +8, March 28, 1844, but no other above the limit appears till July 8th at the same solar meridian. For this reason the succession mentioned merits particular consideration. If we neglect the two cases of July 31 and August 26, which commence at +6 and +5 respectively, we have five cases of successive solar rotations in which the diminutions of intensity began on the +8 day*. This exact recurrence at the end of 26 days of the marked diminution of force proves, it seems to me, *that the actions are all due to the sun*, whose time of rotation must be very nearly 26 days.

An examination of the projected means will show that the sudden diminutions of terrestrial magnetic force are in nearly every case preceded by a sudden increase. It seems extremely difficult to explain these regular pulsations, which are felt all over the earth†, by any theory of distribution of magnetism on the solar surface, or by any variations of temperature on the earth, or to understand why the sudden increase should have ended and the rapid diminution of magnetic force should have begun exactly when the solar meridian +8 arrived at the same position *relatively to the earth*, without admitting that the earth itself had some part in their production—a part which might be due, however, merely to her passage through some ray-like, electrical emanation from the sun.

Lunar Disturbing Action.

Has the moon, then, no part in the production of these disturbances? Since the earth's magnetic force varies during the solar rotation, we may suppose that the moon's magnetic intensity varies from the same cause; but we can scarcely conclude that a change of the moon's intensity with a period of 26 days could produce the comparatively large results found for periods of 29·5 and 27·3 days. I have shown elsewhere that the moon's action on the *diurnal* variation of the *easterly* horizontal force near the equator depends on the sun's position. The moon when on a given terrestrial meridian draws the north end of a needle to the east when the sun is furthest north, and to the west when he is furthest south; the lunar action on the needle is much the greatest during the day (whether the moon be above the horizon or below it), and the lunar

* See Plate 38, where the diminutions in the latter half of 1845 at the +8 meridian are indicated by thick lines. It will be seen that considerable diminutions of intensity occurred near the +8 meridian from the beginning of 1844 to the end of 1845, but that marked diminutions near the zero meridian began in the last days of 1844.

† See plate xxvii. Trans. Roy. Soc. Edinb. vol. xxii., where the variations are given for Makerstoun, Trevandrum, Singapore, and Hobarton. It should be remarked that the projections in that plate are for the Göttingen *astronomical* day, whereas in the Plate illustrating this paper they are for the Makerstoun *civil* day.

action is inverted at sunrise*. We can then only suppose that the moon may attract (or repel) the electric medium through which the solar disturbances are propagated.

If the recurrence of great terrestrial magnetic changes at given solar meridians holds for every rotation of the sun, it is obvious that they should happen equally for all the different positions of the moon; this constant recurrence, however, does not exist, and it is possible that this may be due to the particular positions of the moon, or that the position of the moon may determine the action on the earth. In order to examine this, the same 28 cases were arranged relatively to the moon's positions with reference to the sun and to the equator.

It was found that on the whole more of the large disturbances occurred when the differences of the sun's and moon's longitudes were near + or −90°; but the excess in the quadrants having these differences for the middle points is not sufficiently great alone to give any considerable weight to the conclusion that the disturbances happen preferably near these positions. It may be remarked, however, that the four greatest disturbances during the two years, namely those on March 28 and October 19, 1844, and April 12 and December 3, 1845 (Nos. 1, 9, 15, and 27), in which the mean daily horizontal force diminished from 0·0027 to 0·0036 of its whole value, occurred when the moon was nearly in her first quarter.

On the other hand, when the cases are examined with regard to the moon's position in declination, there can be no doubt that there was one position of the moon for which the disturbances were most frequent, as may be seen in the following Table (p. 402).

The diminution of magnetic force began in eleven cases when the moon was within two days of the greatest northerly position; there were three cases within three days of the greatest southerly position (see Table V.). When the cases are arranged in groups having nearly the same declination, we find

15 cases,	mean declination	beginning	18°·2 N.,	ending	14°·0 N.
5 ,,	,,	,,	12·4 N.,	,,	16·4 N.
3 ,,	,,	,,	15·3 S.,	,,	8·3 S.
3 ,,	,,	,,	12·0 S.,	,,	17·0 S.
2 ,,	,,	,,	2·5 S.,	,,	7·0 N.

There were therefore 20 cases for which the diminution of intensity began when the moon was on the average 16°·7 N., and which ended when she was 14°·4 N. There were only 6 cases for the moon wholly south of the equator. The majority of cases began immediately *after* the moon had attained her greatest north declination (21° nearly).

Since the moon took nearly the same time to pass from the equator to 15° N. as from 15° N. to the greatest northerly position, I have sought for which of these two intervals the cases were most frequent. It appears that of the 20 cases which occurred with the moon north of the equator, 13 happened when her mean position (from beginning to ending of the diminution of intensity) was 18°·7, and 7 when she was 10°·2, so that

* Trevandrum Observations, vol. i. pp. 117, 121, 123.

TABLE V.—Moon's Positions in Declination and in Latitude in each case.

No. of case.	Days from Moon furthest north. Diminution commencing				Moon's mean latitude.			
					Greater than 3°·0.		Less than 3°·0.	
	−9 to +11.	−2 to +2.	+3 to +5.	Other days.	South.	North.	South.	North.
	d	d	d	d				
1.	......	......	+ 3	......	−3°·0 to −4°·5			
2.	......	......	+ 4	......	−3·8 „ −4·5			
3.	......	+ 2	......	......	−2·8 „ −4·4			
4.	......	......	......	− 4	....	+3°·8 to +2°·9		
5.	......	......	......	− 8	....	+5·1 „ +4·9		
6.	......	0	......	......	....	....	−1°·0 to −2°·0	
7.	......	......	......	− 8	....	+5·0 to +4·0		
8.	......	− 2	......	......	....	....	....	+1°·3 to +0°·3
9.	......	......	......	− 9	....	+4·5 to +5·2		
10.	......	......	......	− 7	....	+4·9 „ +3·7		
11.	......	......	+ 4	......	−4·5 to −5·1			
12.	−10	......	......	......	....	+3·7 „ +5·0		
13.	......	− 1	......	......	....	....	+0·1 to −1·9	
14.	......	......	+ 5	......	−4·7 to −5·0			
15.	......	+ 1	......	......	−2·6 „ −4·2			
16.	......	......	+ 5	......	−5·2 „ −5·0			
17.	......	......	......	− 5	....	....	....	+3·4 „ +2·4
18.	......	+ 2	......	......	−3·5 to −4·9			
19.	......	+ 1	......	......	−2·6 „ −3·5			
20.	......	......	+ 4	......	−4·6 „ −4·9			
21.	......	+ 2	......	......	−4·2 „ −4·7			
22.	−10	......	......	......	....	+4·4 to +5·2		
23.	......	+ 1	......	......	−3·3 to −4·1			
24.	+11	......	......	......	....	....	−1·5 to +1·1	
25.	......	0	......	......	−2·0 to −4·5			
26.	+12	......	......	......	....	....	....	−0·7 „ +1·8
27.	−10	......	......	......	....	+5·2 to +5·2		
28.	......	− 2	......	......	....	....	−0·6 to −2·7	

the more northerly cases were the more frequent. In the same way it is found that of the 6 cases for which the moon was south of the equator, in 4 the mean declination was more than 15° (=15°·6) and in 2 less (=10°·8). There were thus only 2 cases remaining for which the mean declination of the moon was within the limits 10°·2 N. and 10°·8 S., a space covering half the moon's whole path.

It would appear, then, that the exact recurrence of the greater diminutions of terrestrial magnetic force for certain days happens only in the solar period, but that their frequency depends on the position of the moon relatively to the equator. It may also be remarked for the cases Nos. 18, 19, 20, 21, 23, 25, and 28, for which the disturbance commenced near the solar meridian +8, the moon was within two days of her greatest northerly declination (except one case, for which her mean declination was 15° N.).

It has been supposed that this apparent action of the moon is due to her position relatively to the *equator*; but as the nodical period differs only 0·11 day from the tropical, it is possible that it is her position relatively to the *ecliptic* which is in question. I have in consequence examined the cases with regard to the moon's latitude, with the following results (see Table V.). Noting that the greatest latitudes are between 5°·0 and 5°·3, we find

13 cases, mean latitude beginning	3°·6 S.,	ending	4°·6 S.
8 ,, ,, ,,	4·6 N.,	,,	4·5 N.
1 case ,, ,,	3·4 N.,	,,	2·4 N.

6 cases beginning and ending within the limits 3·0 S. and 3°·0 N.

There were 21 cases for each of which the moon's mean latitude was greater than 3° S. or than 3°·3 N.; 1 case for which the mean latitude was 2°·9 N.; and 6 cases for which the mean latitudes were between 1°·7 S. and 0°·8 N. The relative amounts of the diminutions of magnetic force are projected on curves supposed to represent the moon's path in declination (fig. A) and in latitude (fig. B, Plate 38).

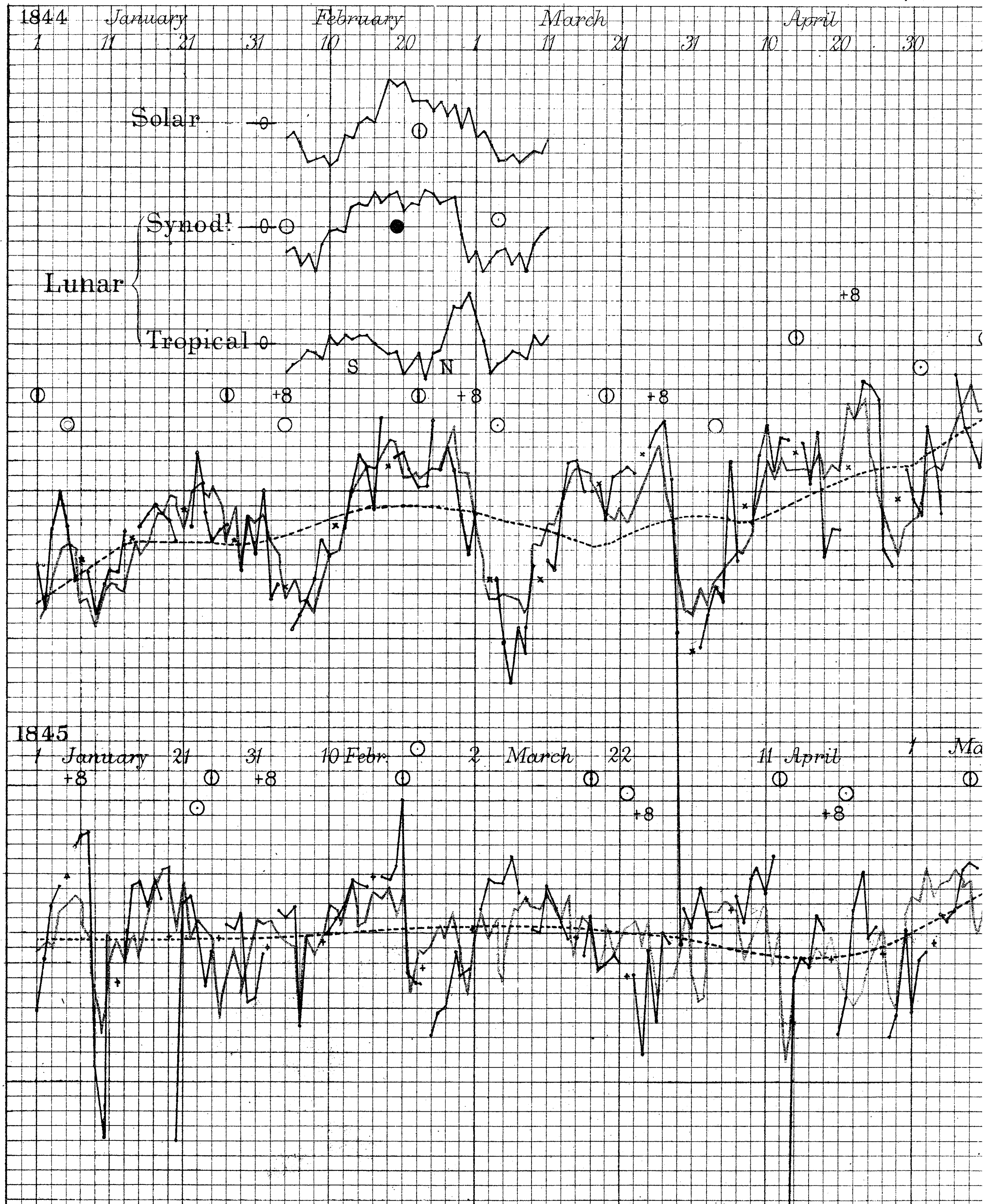
It will be seen from these two figures that 13 cases occurred for the moon when passing from the greatest northerly declination towards the equator, while for the passage below the ecliptic there were 16 cases. From near the greatest *southerly* latitude (fig. B) to near the greatest northerly latitude (embracing 11 days of the whole period) there were only 2 cases. The proportion of cases for the two portions of the moon's path was as 26 : 2, whereas the ratio should have been as 16 : 12 had the chances of disturbance been equal for all parts of the moon's orbit.

There is no doubt, then, that in 1844 and 1845 the greater disturbances of mean magnetic force occurred most rarely when the moon was near the equator or ecliptic; but it is not possible to determine from two years' observation whether the moon's action depends on her position relatively to the one or the other plane only, or to both. If *the thing acted on*, whatever that may be, *retains the same position* relatively to these planes, the discussion of 5 or 6 years' observations will show to which plane these results should be referred.

The preceding investigation was limited to cases for which the diminution of daily mean horizontal force within three days was not less than one thousandth of its whole value; the change of daily mean horizontal force from day to day (without regard to sign or limit) has also been sought for the solar and each of the lunar periods. The results confirm the conclusions already derived from the greater negative movements, as to the solar meridians and moon's latitudes for which the disturbance is a maximum. I shall defer, however, the consideration of this part of the investigation till a longer series of observations has been discussed.

Broun.

Daily means of the horizontal Force of the Earth's Magnetism for th



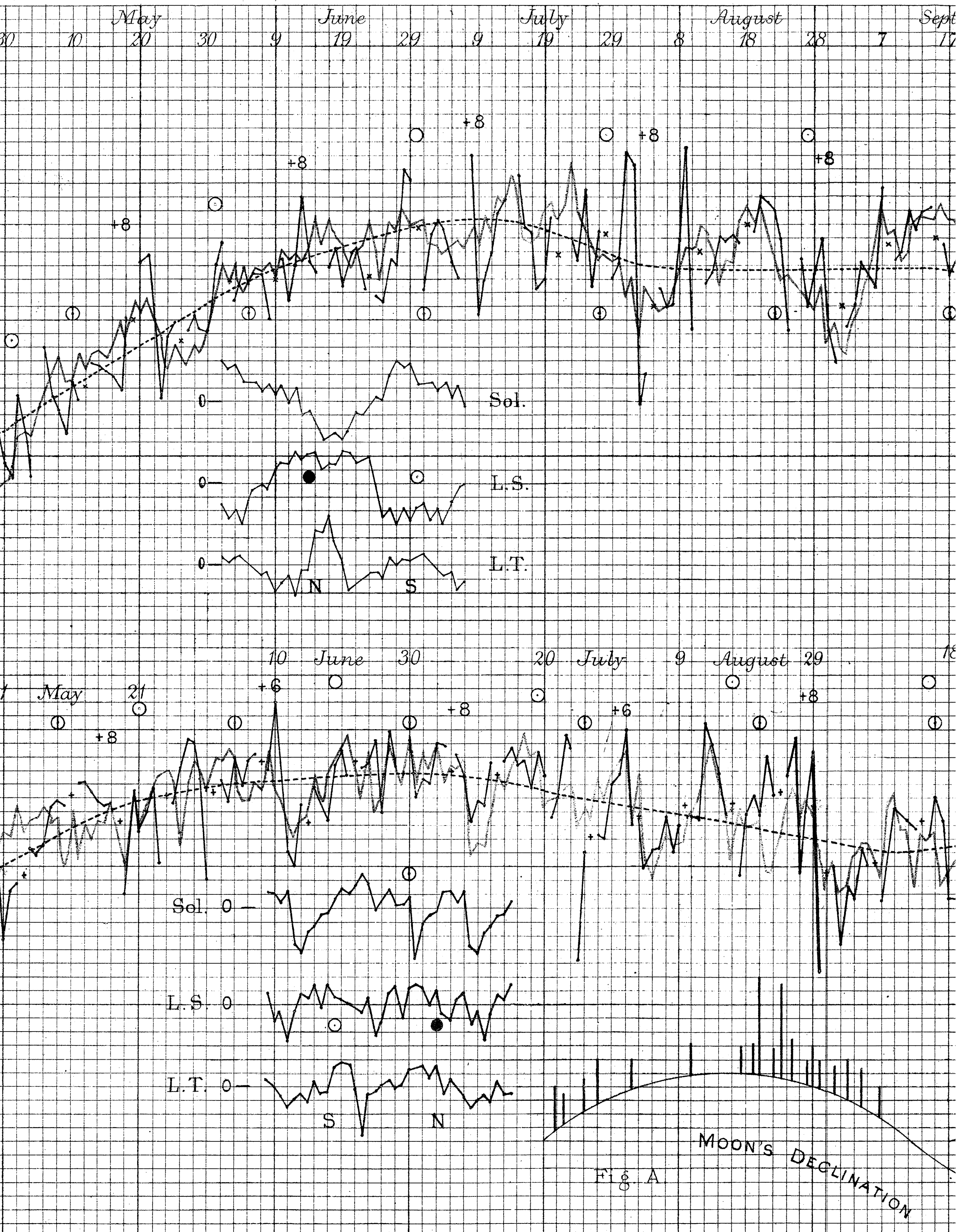
One Inch = one thousandth of the whole horizontal Force = $\frac{x}{1000}$

The means are for the Civil day.

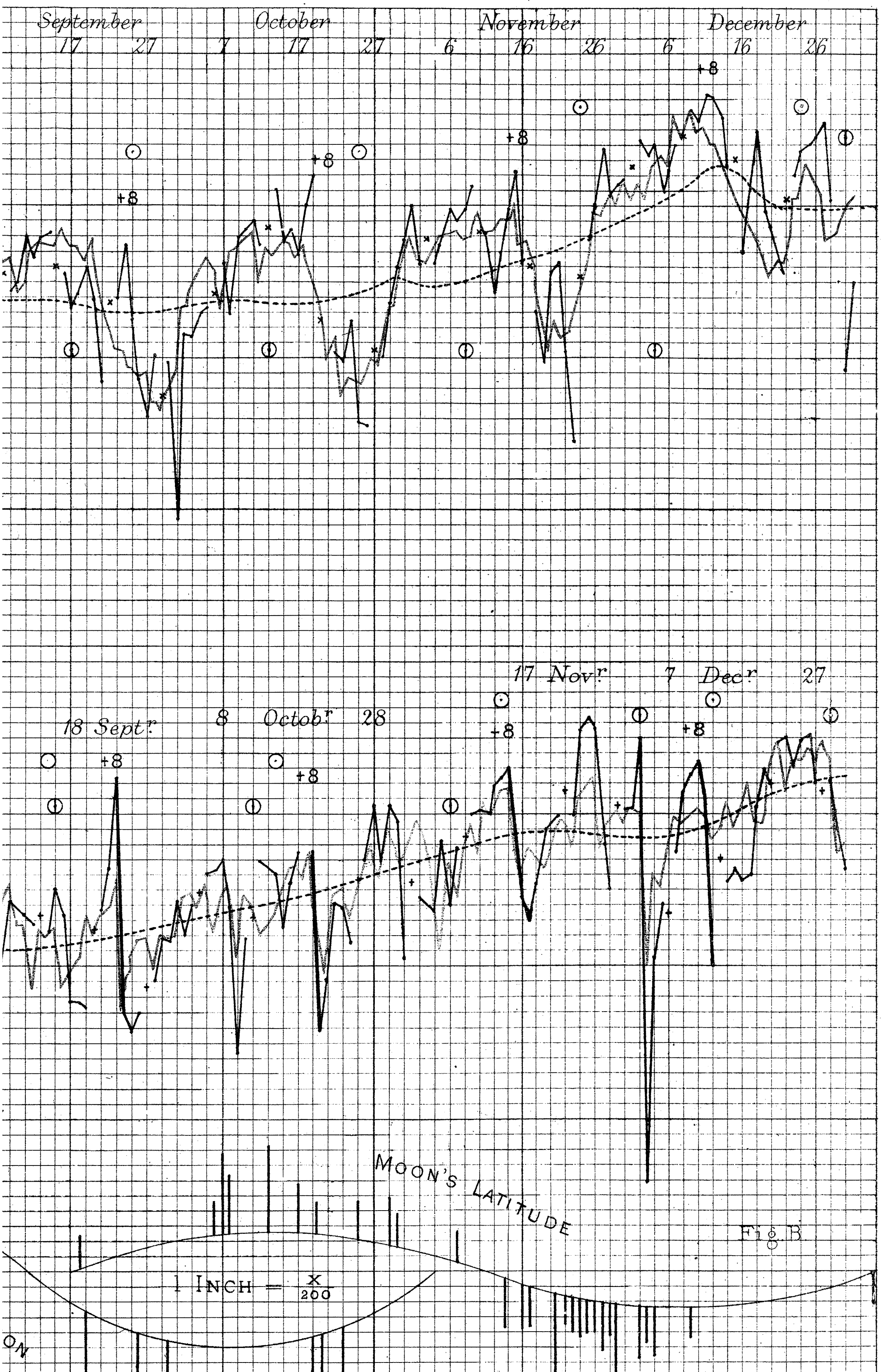
○ *Full Moon* ● *New Moon.* 0 — *Zero line.*

N. *Greatest North*, **S.** *Greatest South declination.*

for the years 1844 and 1845, as deduced from the Makerstoun Observations



…tions (black lines) and as calculated (red lines) by J. A. Broun.



○ *Full Moon* ● *New Moon.* 0 — *Zero line.*

N. *Greatest North*, **S.** *Greatest South declination.*

⦶ *First or Zero Solar Meridian.* +8 *Meridian.*

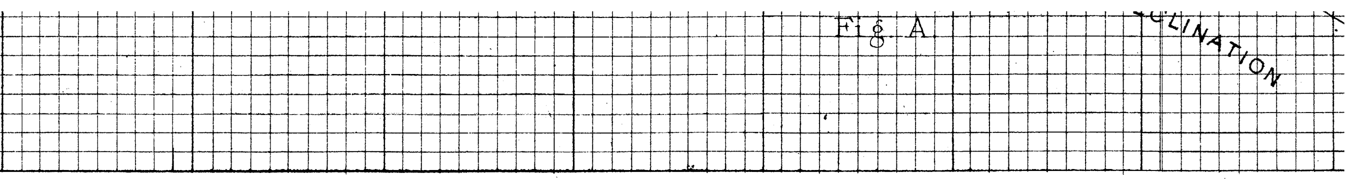
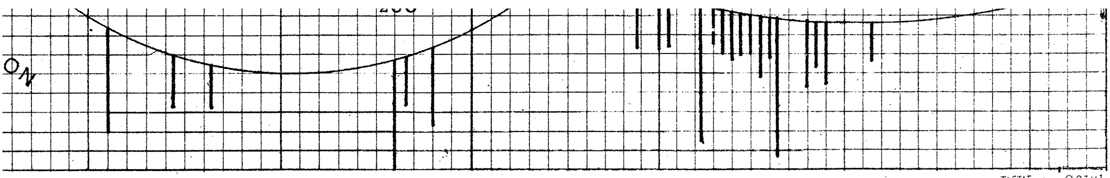
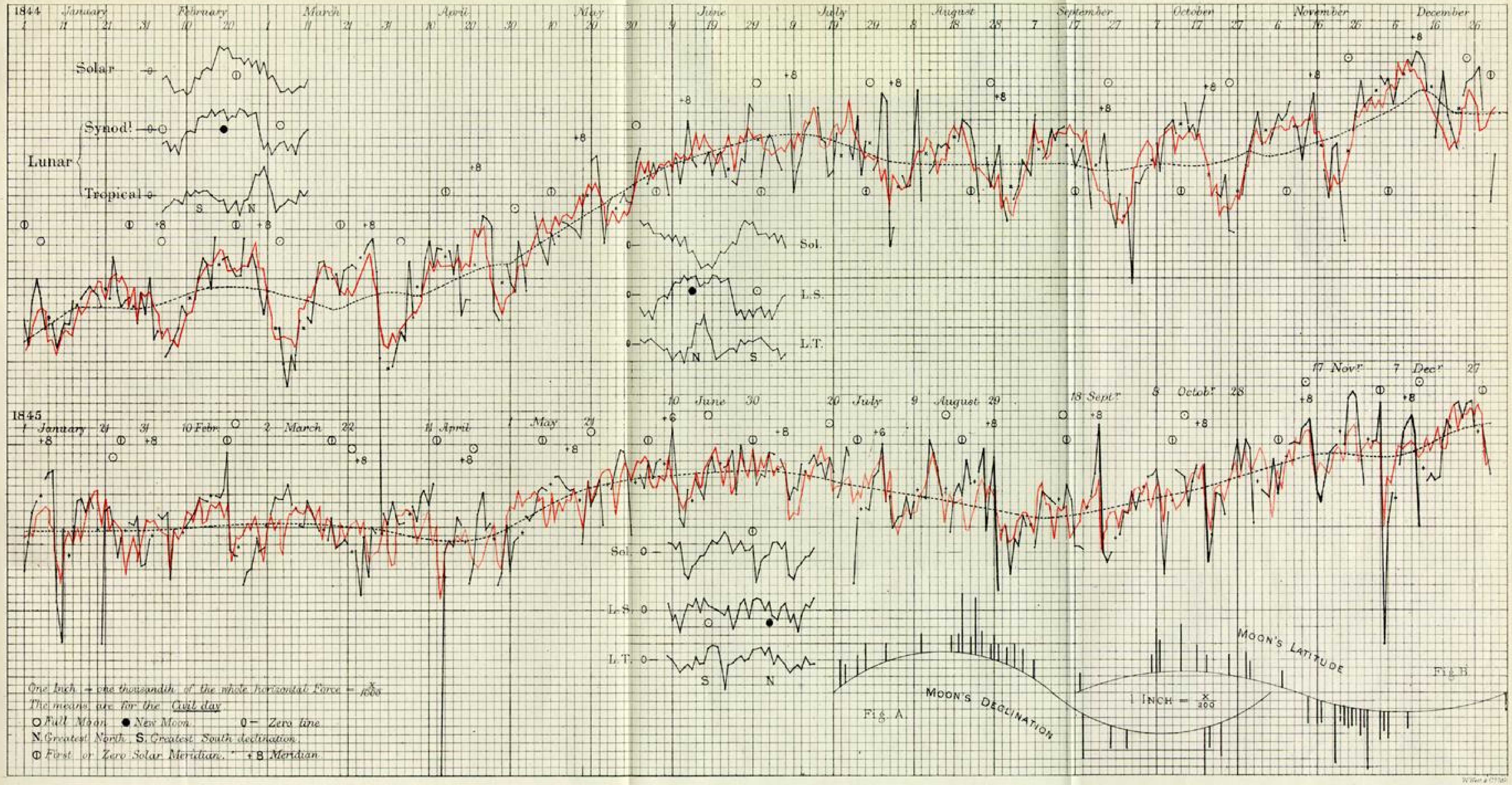


Fig. A



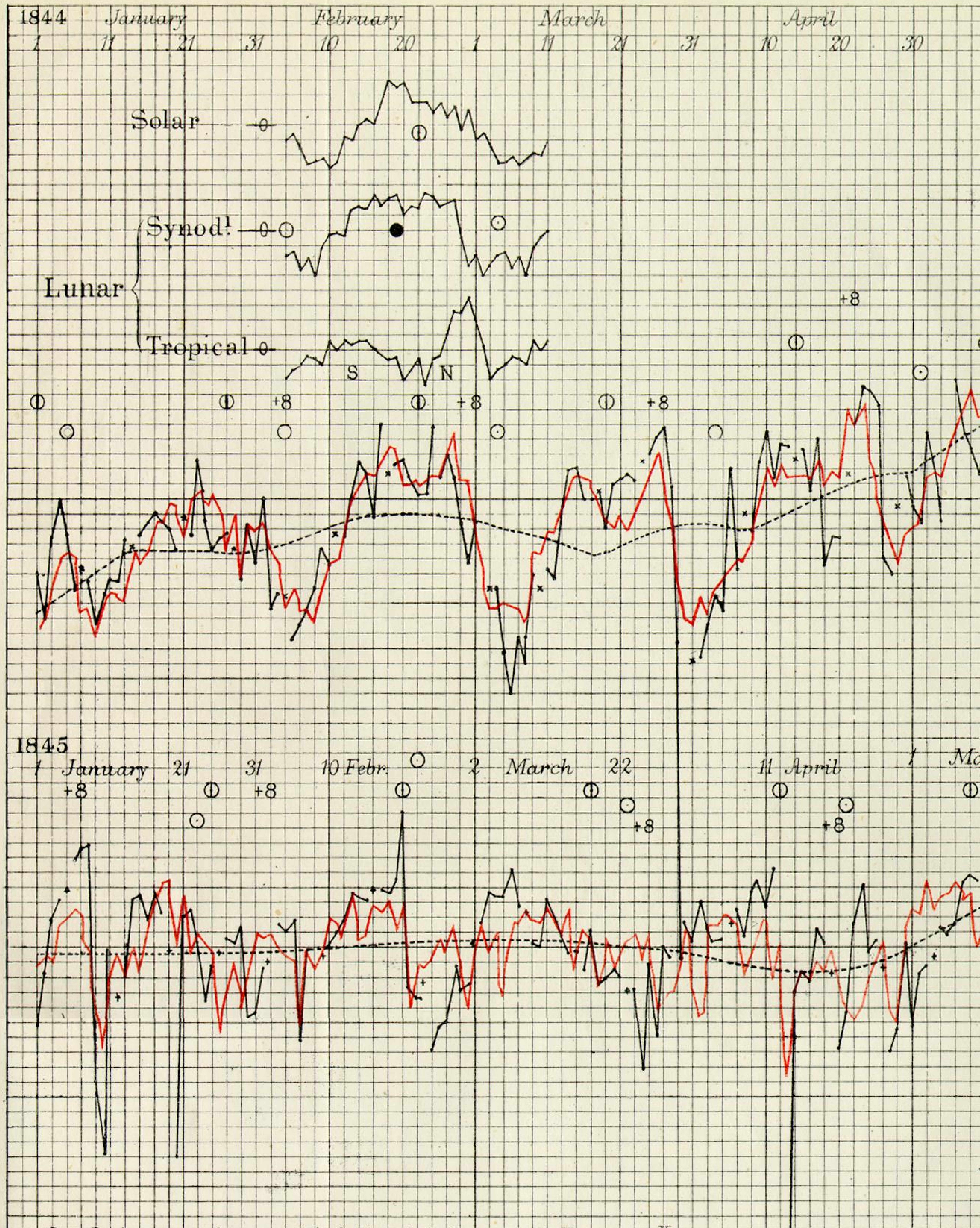
W. West & C° lith.

Daily means of the horizontal Force of the Earth's Magnetism for the years 1844 and 1845, as deduced from the Makerstoun Observations (black lines) and as calculated (red lines) by J. A. Broun



Broun.

Daily means of the horizontal Force of the Earth's Magnetism for th



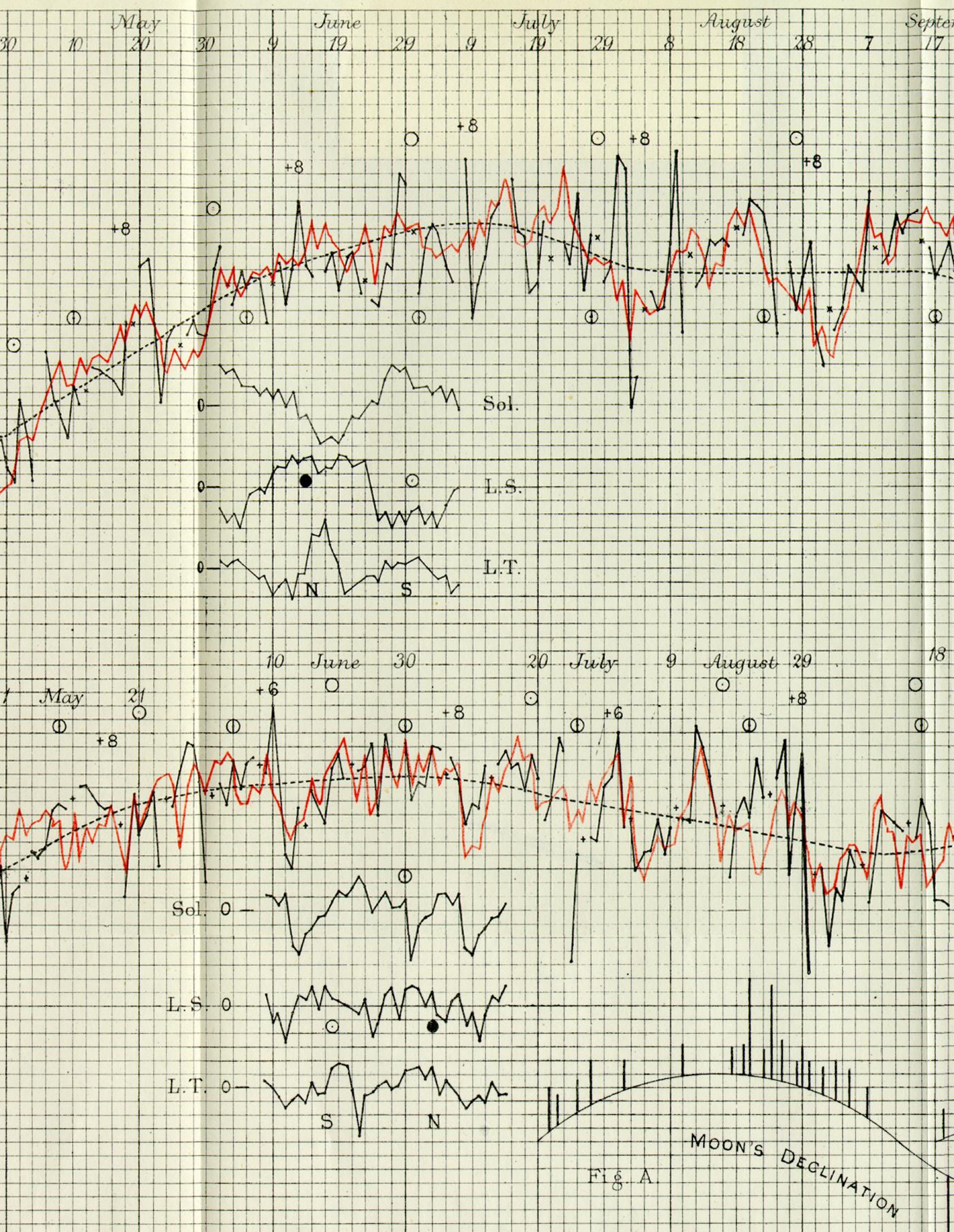
One Inch = one thousandth of the whole horizontal Force = $\frac{X}{1000}$

The means are for the Civil day.

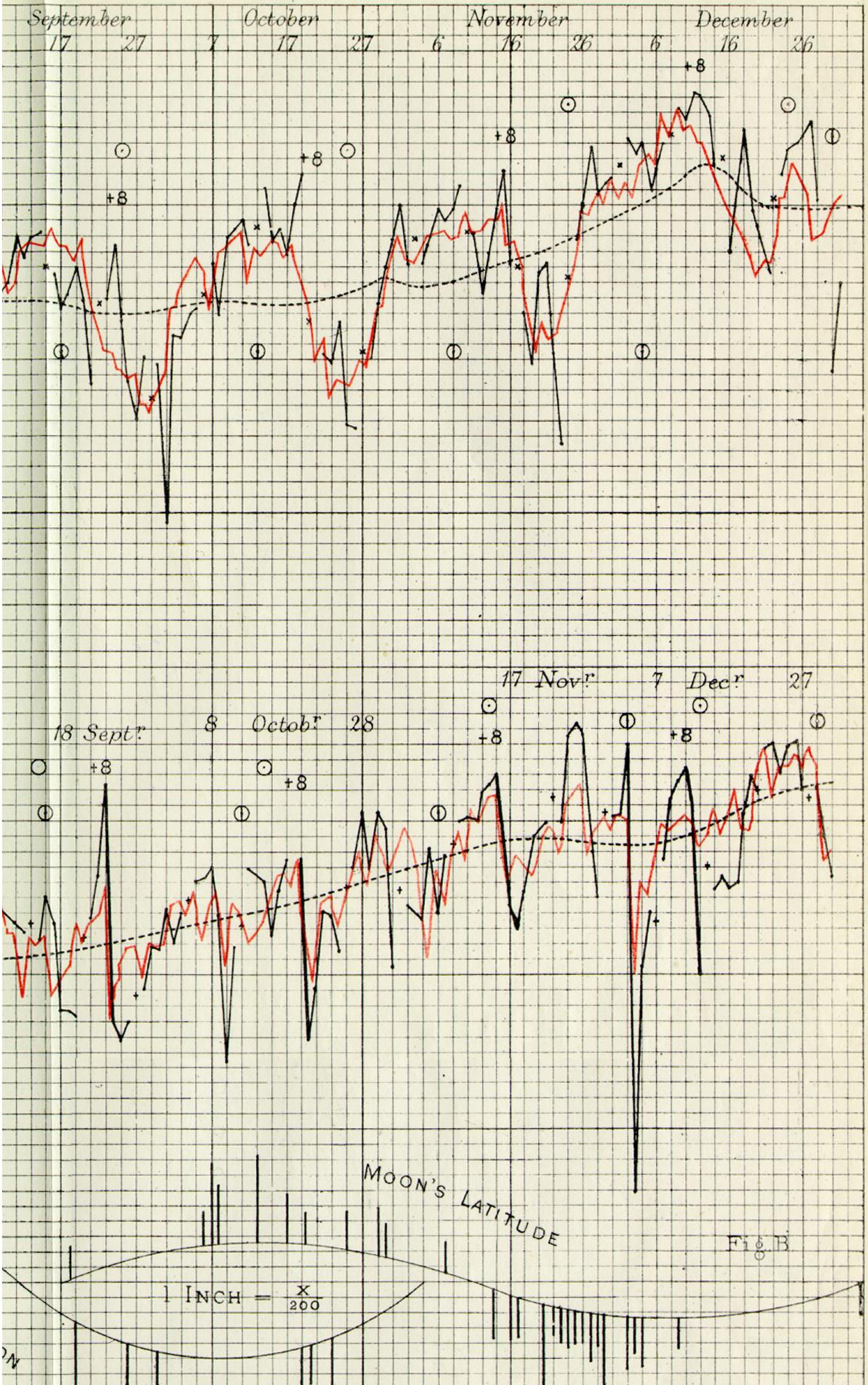
○ *Full Moon* ● *New Moon.* 0— *Zero line.*

N. *Greatest North*, **S.** *Greatest South declination.*

for the years 1844 and 1845, as deduced from the Makerstoun Observations



tions (black lines) and as calculated (red lines) by J. A. Broun.



○ *Full Moon* ● *New Moon.* 0 — *Zero line.*

N. *Greatest North,* **S.** *Greatest South declination.*

⦶ *First or Zero Solar Meridian.* + 8 *Meridian.*

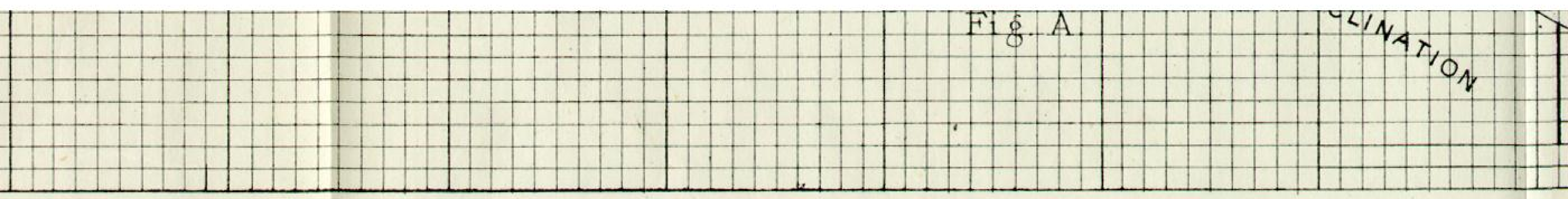
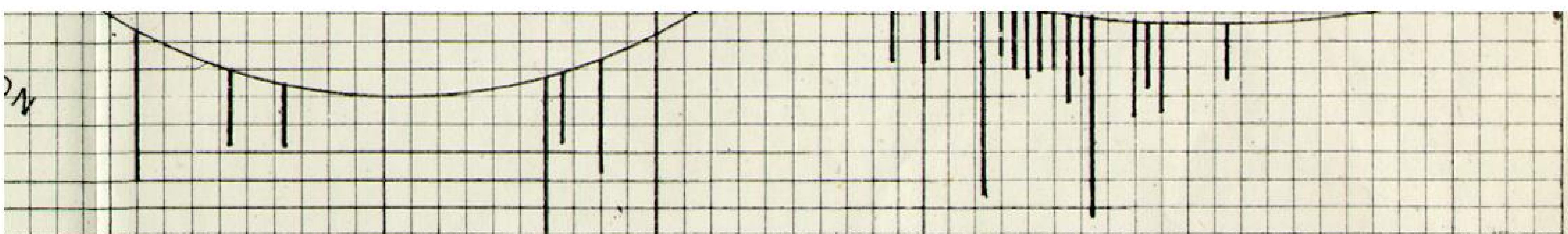


Fig. A.
LINATION



W. West & C° lith.